Simulated and Observed Scaling in Earthquakes

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Abstract Earthquakes do not fit into the class of models we discussed in Physics 219B. Earthquakes and fault systems instead are an example of a self-organized critical system. In this paper I will explain the Virtual California earthquake simulator, and show that the simulated fault system in California displays power law correlations exemplary of critical behavior.

1 Self-Organized Criticality

Earthquakes are believed to be in a class of complex dynamical systems called self-organized critical phenomena. In simple dynamical systems with few degrees of freedom, power law correlations can arise when one fine tunes a parameter to arrive at a critical point. But for dynamical systems found in nature there are no scientists running about tuning various knobs, so how can apparent critical behavior arise?

Research on the statistical properties of these dynamical systems in the late 1980s, pioneered by Per Bak, has shown that certain interacting dynamical systems — called self-organizing critical systems — naturally evolve into a statistically stationary state. This state is also critical, with power law spatial and temporal correlations. It is essential that these systems are dissipative and are spatially extended with an innumerably large number of degrees of freedom. Energy must be fed into these systems in a uniform manner, either into the bulk or through the boundaries. The earth's crust, subjected to forces from tectonic plate motion, may be viewed as a kind of self-organizing critical system [1].

For the fault systems in the earth's crust, this self-organized critical state is a balance between local geologic forces, which adjusts the probability that a slip on a fault segment will propagate to a neighboring segment close to unity. The probability of this activity branching to neighboring fault segments is balanced by the probability that the slip has sufficiently reduced the local stress. This stationary state is often interpreted as a critical chain reaction. At this critical state there is no characteristic time, space, or energy scale, and all spatial and temporal correlations are power laws. For earthquakes the power law size distribution is directly related to the fractal structure of fault systems. The main assumptions from self-organized critical phenomena applicable to the study of fault systems are that the system is large and that the driving force, in our case tectonic plate motion, is slow.

2 Power Laws and Scaling in Seismology

In 1954 the first empirical evidence for earthquakes exhibiting power laws was introduced by Gutenberg and Richter [4]. They put forth the now widely accepted scaling relation between earthquake occurrence and magnitude

$$N(\geq m) = 10^{\,a-bm} \tag{1}$$

where a and b are empirical constants and N is the cumulative number of earthquakes with magnitude greater than m. This is the most widely accepted evidence supporting the idea of fault systems as an example of self-organized criticality. Global seismic data from the Advanced National Seismic System' global earthquake catalog (www.quake.geo.berkeley.edu/cnss/) catalog is plotted in Figure 1 for the years 1990-2010, showing close agreement with equation 1. The best fit parameters were obtained by a least squares fit to the data between $5.5 \geq m \geq 7.5$. The slight deficit below magnitude 5 is attributed to the sensitivity limit of seismic monitoring equipment. The deficit at $m \geq 7.7$ is more controversial. It is usually attributed to the transition in topology from small earthquakes with similar lengths and depths to large earthquakes with lengths much larger than depths [10].

There has been debate as to whether Gutenberg-Richter scaling should be evaluated globally or it should be taken to apply only within regional fault systems [10]. Indeed if fault behavior is due to regional fault geometry, it would seem natural to apply it on a regional basis. The difference in agreement with Gutenberg-Richter scaling on a global or regional scale is seen in comparing Figures 1 and 2.

In 1997 Turcotte [11] showed that the Gutenberg-Richter scaling relation is equivalent to a power law scaling between N and the earthquake rupture area A_r:

$$N \sim A_r^{-b} \tag{2}$$

Equation 2 provides further evidence that earthquake behavior is ultimately tied to local fault geometry. This correlation is measured from simulations in Figure 3.

Omori's Law is another example of seismological power law correlation between frequency and magnitude of an earthquakes aftershock. The law essentially states that for every magnitude 8.0 earthquake there will be 10 magnitude 7.0 aftershocks, 100 magnitude 6.0 aftershocks, 1000 magnitude 5.0 aftershocks, etc. Following a large earthquake, likely above the level predicted by the relation in equation 1, it is believed that a series of smaller earthquakes act to fill in the rest of the magnitude range to push the system back into long term compliance with equation 1, [10].

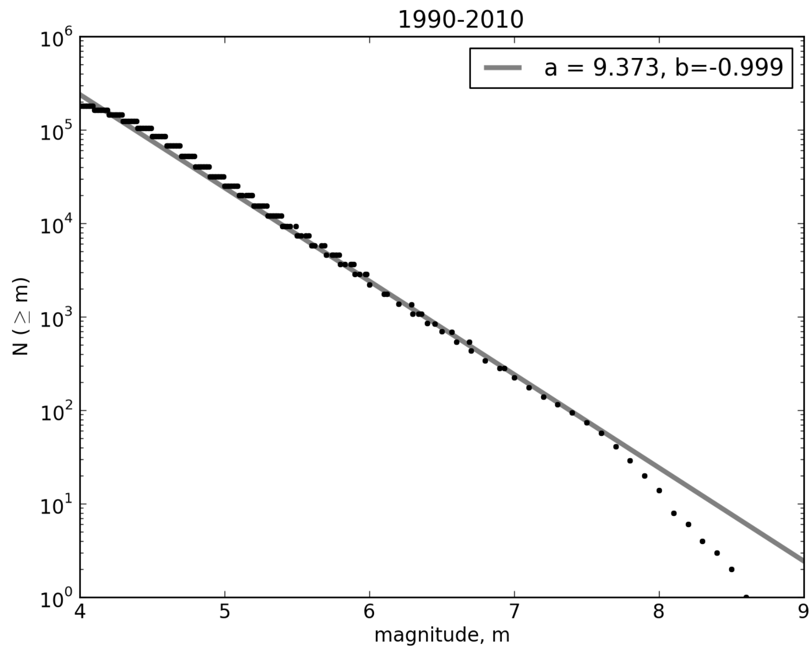

Figure 1: Global seismic data from 1990-2010. Best fit line to equation 1 for $5.5 \geq m \geq 7.5$ is shown in grey.

In the following sections I will introduce the relationship between earthquake simulations and statistical mechanics. Then I will outline the simulation program that I use in my research — Virtual California — and show that it fits the definition of a self-organized critical system. Lastly I will show how observed California and global seismicity compare to ensemble averages of the simulated fault systems, showing that indeed the simulations accurately reproduce the complex dynamics of an interacting fault system.

3 Simulations in Ensemble-domain vs. Time-domain

Earthquake fault simulations fall into two general categories: time-domain simulations and ensemble-domain simulations. Time-domain simulations are highly analytical, taking input fault parameters current states then solving the governing differential equations using finite-element methods to evolve the system in time to develop a time dependent fault model. Besides being incredibly computationally intensive, the output from time-domain simulations are quite sensitive to small perturbations in the initial conditions. Furthermore, to be applied over any spatially extent region, it requires extensive field measurements of fault parameters and current fault conditions.

An ensemble-domain simulation avoids such problems by taking only a few observed fault parameters and then looking for the most likely future states of the system. The output is then not a deterministic time evolution of the system, but rather it is a single realization obtained by a sort of importance sampling from the ensemble of possible future states of the fault system. A familiar example of this approach is the Metropolis Monte Carlo algorithm. Virtual California is an example of an ensemble-domain simulation whose output from each simulation is a member of an ensemble in configuration space.

4 Virtual California Earthquake Simulator

Virtual California is a computer program that simulates topologically realistic driven earthquake fault systems in California [7, 3, 8]. Virtual California is designed to quickly simulate many thousands of events over long periods of simulated time, producing a rich dataset to study the statistical properties of the fault system (described in detail in [8]). Essentially based on the slider-block model popular in the fields of complexity and statistical mechanics, the simulator consists of three major components: a fault model, an interaction model and an event model.

4.1 Fault model

Despite what the name implies, the only part of Virtual California that is specific to California is the fault model. Fault topology, long-term slip rates, and frictional parameters are derived from field observations to define the fault model. The model that is currently in use is based on the Uniform California Earthquake Rupture Forecast version 2, described in [2]. The model consists of 65 major fault sections that roughly correspond to the known fault system in California, with some larger faults modeled by multiple sections (e.g. the San Andreas fault). The fault plane in each section is meshed into square elements that are approximately 3km x 3km, for a total of 8395 elements. Each element is given a stress threshold and a slip velocity along a fixed vector, obtained from geologic field-measured values. The failure stress is an inherently unmeasurable quantity, and is tuned such that the simulation reproduces observed earthquake recurrence times for that fault section.

4.2 Interaction model

Interactions between different fault elements are governed by Okada's quasi-static elastic half-space Greens' functions [6]. The effect between elements (not only nearest neighbors, long range interactions are allowed) depends on their relative position and orientation and on the direction of their slip displacements. The interactions are quasi-static because the rate of change of the fault geometry is extremely slow. The average slip velocity is on the order of 2x$10^{-10} m/s$, meaning that over 10,000 years of simulated time the average slip rate produces about 50 meters in displacement which is less than 0.1% of the size of the fault element.

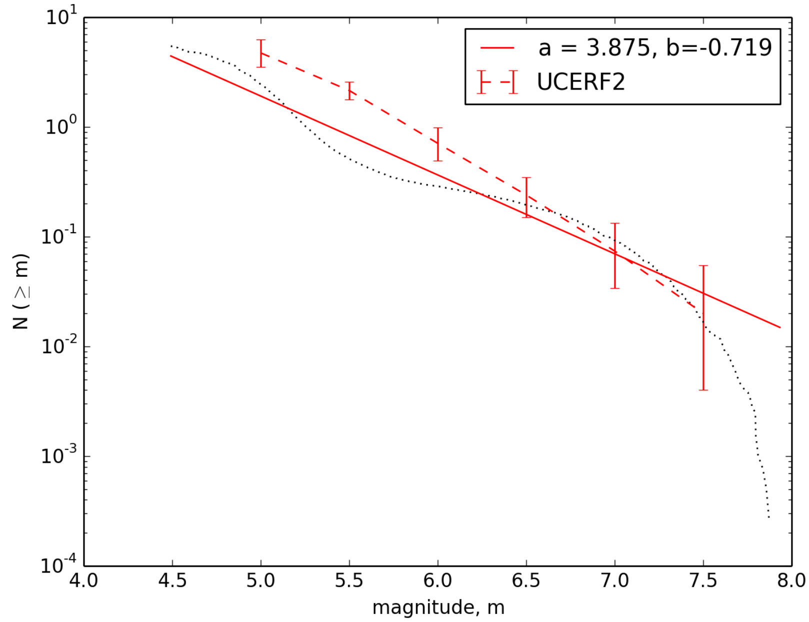

Figure 2: An average of 20 realizations of 200 years of California seismic history. Best fit line to equation 1 for $5.0 \geq m \geq 7.5$ is shown in grey. The observed seismicity for California with uncertainty given by UCERF [2].

4.3 Event model

"Backslip" is applied to the elements at geologically-observed rates. Backslip does not cause the elements to move, but loads stress on the element because of the accumulation of a slip deficit (a distance it "should have" moved given the slip rate and duration). This continues to build stress on the elements until the frictional parameters (coefficient of friction) are exceeded, then the segment actually begins to slip. The slip continues to drop the stress on the local element and transfer it to the surrounding elements via the elastic Greens' functions until a residual stress is reached, plus or minus a random overshoot or undershoot of typically 10%. The transferred stress results in propagating ruptures (groups of elements slipping together) throughout the system, a simulated earthquake (described in detail in [3, 5]).

5 Simulations vs. Observation

I used 20 Virtual California simulations each of 200 years of seismic history to approximate the ensemble average for comparison to equation 1 for California earthquakes. I also compared this average to the observed seismicity in Califor-

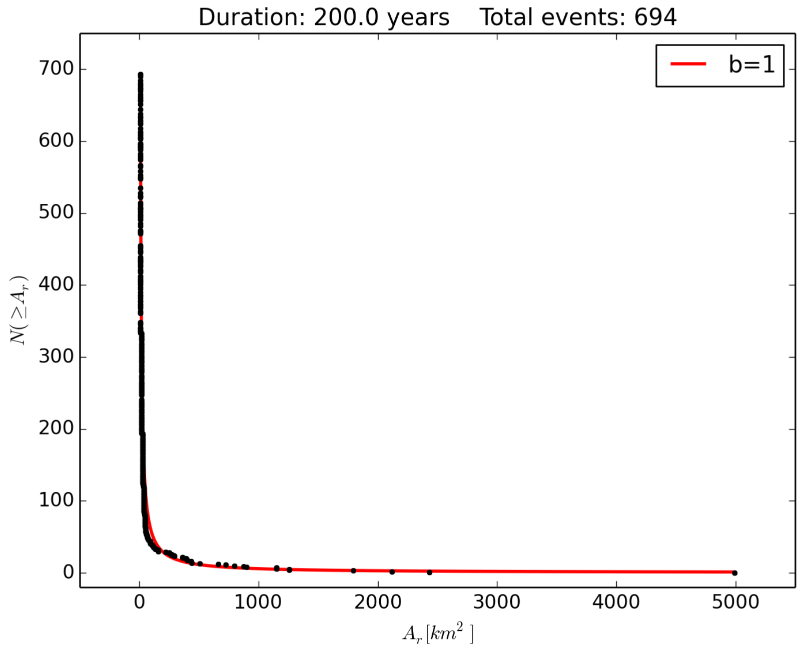

Figure 3: A match to equation 2 from a single 200 year simulated seismic history for California. Equation 2 plotted in red.

nia as reported by [2], shown in Figure 2. As shown, the simulated ensemble average closely matches the observed seismicity in California. The deficiency of earthquakes below magnitude 6.0 is a result of the finite size of the fault elements. With a finer grid resolution, the fraction of smaller earthquakes would increase.

Shown in Figure 3, I was able to reproduce the relation in equation 2 with a single 200 year simulation. The previous figures show that indeed the simulated fault system is in a self-organized critical state. Furthermore the similarity of the simulations to observed seismicity demonstrate the power of statistical mechanics in shedding light on such complex natural processes.

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